The construction of a problem: Architecture modelling after Descartes

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Architectural design and modelling has switched its focus from object representations of design resolution to constructing the design as a system for interrogation. This can be seen as a late shift from an architectural foundation of classical geometry to the uptake of modern analytic geometry in architecture. It is an exciting creative development but it also poses some new and explicit spatial challenges for working in much more complex design model spaces. These challenges manifest in three architectural modelling spheres: the conceptual, the operational and the constructional. How do architects cope with a representational medium that runs to many dimensions and invites models that cannot be viewed or visualised, and what does spatial syntax have to do with navigating and interrogating architectural model spaces like these? Syntax belongs to language. It is arguably a high level imposition on space but it is useful in the symbolic representation and manipulation of geometry. This paper briefly explores the challenges of the architectural shift to modern geometry. It does this through philosophical framing and hands-on architectural examples in each of the conceptual, the operational and the constructional spheres of architectural modelling. It looks for some clues, if not conclusions, about appropriate space syntactic approaches to the architectural model.

1. Introduction

Object making in architectural design has been progressively replaced by engagement with systems. Digital computation and the increasing power of computers have accelerated this process and brought systems and the modelling of variables and relationships into prominence. The development of electronic computation in no way marks the birth of the underlying ideas or thought processes around the systemic. Architecture, like linguistic, literary, musical, numerical and notational systems is a manifestation of the human propensity to construct such pattern systems. But in the context of this paper, the growth of a focused interest in systems as the basis for architectural design representation dates from the post World War II period in concert with the development of electronic computation.

In the systems paradigm, both the ‘raw ingredients’ and the organisational patterns of architecture remain substantially geometrical, albeit with the opportunity to link to many non-geometrical attributes. In this paper, I argue that there has been a late coming of age in architecture with the embrace of modern as well as classical thinking with respect to geometrical organisation and form making. Ironically ‘modernism’ in architecture, despite its pluralist retrospectives in the nineteenth century, orchestrated a renewed commitment to a classical approach to geometry in the twentieth century.

At one level there has been an increased aesthetic exploration of the application in design of ideas that have been articulated in mathematics and the sciences in the modern era. These are specific concepts such as chaos, complexity theory, various families of surface, non Euclidean hyperbolic and Riemannian geometry, and topology.

At a deeper level there is a progression from the Classical appreciation of geometry as a system of relationships given or pre-existing in the world, to the later Modern appreciation of geometry as a way to ‘construct a problem’ (Lachterman, 1989).
The shift from classical to modern geometrical thinking

Greek geometry is explicated through constructing the geometry itself. In architecture, this is the relationship characterised by Evans (1995, p.xxvi) when he writes:

‘The elements of geometry are thus conceived as comparable to the bricks that make up a house, which are reliably manufactured elsewhere and delivered to site ready for use. Architects do not produce geometry, they consume it.’

After Descartes and the seventeenth century, modern geometry becomes a more exploratory activity that embraces a humanist virtuosity in making or construction, and implies mindful Will in the directions this takes. This progression from classical to modern geometrical thought and activity is a more contemporary right of passage in architectural design. The spirit of exploration of relationships of context, form, structural and constructional constraints, and performance-drivers inherent in computational ‘form finding’ moves the traditional artisanal practice of architectural design away from emulating and refining the classical construction of geometry and towards Descartes’ own ‘construction of a problem’. Lachterman (1989, p.vii) argues: ‘A fairly direct line runs from the “construction of a problem” (Descartes), through the “construction of an equation” (Leibniz) to the “construction of a concept” (Kant).’ Now we might add, only slightly tongue in cheek, the ‘construction of a design’ in architecture.

Modern geometrical construction emphasises the activity of creation itself; the central ontological question is the activity of construction as the proof of the existence of mind. Constructability gives geometry its value but as a system for further making, for further feats of the mind. This is in marked contrast to the constructability in Euclid’s Elements that gives the proof of existence of the mathematical entities themselves (Lachterman, 1989). Thus, the transition from the emphasis on the given object in geometry to the actively engendered generative system in mathematics is seen to occur in Descartes’ work (ibid., p.xiii).

The uptake of this modern appreciation of space through analytical geometry presents new and profound complexities to designers and design modellers in three different spheres: the conceptual (the model), the operational (modelling) and the constructional (making). The shift increases the creative power and vocabulary of the designer but also leads to relinquishing control in terms of known geometrical consequences. In this respect it is possible to construct spaces that are challenging to navigate even for the creator. This paper considers these navigational challenges in the conceptual, operational and constructional architectural design spheres and opportunities for revisiting geometry and its philosophy for approaches to the navigation of architectural design model spaces.

2. Architecture and space syntax

The biological analogy and implications for the artificial

Like Lachterman’s analysis of Descartes’ work, Hillier and Hanson (1984) might also be said to have been ‘constructing a problem’ as they articulated their socially created analysis of constructed space. In challenging the man-environment paradigm, they argued that the man-made world is already a social behaviour rather than shaping one. Extracting geometrical systems from space as social process led to some fundamental principles. They found that existing settlements could be described through a finite set of elementary generators applied as restrictions on a random process, likening this to genotypes, abstract rules underlying spatial forms. The spatial form itself (the architecture) is the phenotype in this system. They note that this simplified their work, as “[t]here were fewer genotypical varia-
tions than phenotypical variations’ (Hillier and Hanson, 1984, p.12). Their work uncovers combinations of randomness at the cellular or phenotypical level with ordering constraints at the genotypical level. They refined this analysis through mining existing settlement patterns. This analysis is represented topologically, applying graph theory to a system of nodes (spaces) and links. It is a thoroughly modern geometrical approach.

In parallel and since that time there has been much exploration of the application of a similar line of biologically analogous systems thinking to the synthesis of the phenotype, or the architecture itself. Generative and parametric systems in which it is the underlying rules of the system that are manipulated rather than the form itself have transformed the process of architectural design. Paradoxically they have tended to lend greater emphasis to form per se. By fostering greater geometrical complexity, this has also enlivened architectural engagement and debate around tectonics. This includes the means to realise in physical space the system relationships that can be modelled computationally. Hillier and Hanson (ibid.) equate the ‘societal investment in order in space’ to the level at which it has been necessary to restrict a random process to arrive at the form. In contemporary architecture the instigation of the random or at least the varied and unpredictable into the geometrical schema for design equates in most cases to a higher order of production challenge and subtler and more considered aesthetic. Thus the relationship is reversed. The societal investment in order in architectural space becomes proportional to the degree of randomness, or release agency permitted in the system, while still holding the genotypical reins at the meta level.

Architectural modelling and space syntax
Space syntax has proved a useful basis of abstraction to analyse and predict social behaviour for future urban spaces. But it is at heart an analytical approach to existing urban structures that seeks to enhance both intuitive pattern recognition and metrical analysis. It does this through breaking down the complexity of the overall emergent city organisation into components whose relationships can be interrogated and mined systematically through computation. A powerful systems-based approach to abstraction and analysis has great value at the level of abstract comprehension of the increasing complexity of massive polycentric and amorphous metropolitan and urban regions.

By contrast, digital computation in architecture provides the opportunity to construct very information rich spatial system models to represent prospective architectural spaces. It is at heart a synthetic approach to representing and interrogating design intent. The graphs of these design spaces can quickly generate innumerable potential geometrical design solutions. In switching attention from the analytic to the synthetic, a many-to-one analysis becomes a one-to-many relationship in terms of the design outcomes of each relationship in the model. Beyond the geometry itself, geometrical parameter values can be linked to other behaviours (social, phenomenal: wind, heat, light, sound, etc.) or to other attributes (materials and properties). Beyond social and architectural space, system modelling makes explicit another complex space: design space or model space. The inherent complexities of this space invite a fresh recourse to the idea of spatial syntax.

Following Kant, geometry is not merely the syntactic symbol system through which we understand and communicate spatial relationships; it is an innate characteristic of space as we construct and understand it, a priori intuition, ‘a science that determines the properties of space’, (Kant, 1981, p.70). It can also appear to lack the semantic fluidity possible in language in thought, conversation and even evocative sketching. It is possible to create in language, natural or logical relationships, which
cannot be immediately brought into existence spatially or geometrically.

**Architectural systems modelling and geometrical syntax**

Just as space syntax developed as a named collection of procedures from the late 1970s to early 1980s, expressly grammatical or syntactical approaches were also tested in art and architecture, albeit synthetically. George Stiny and James Gip's shape grammatical approach to generating two-dimensional art works (Stiny and Gip, 1971), Stiny and Mitchell's Palladian grammar (Stiny and Mitchell, 1978), and Paul Coates 'design grammars' of higher level CAD functions (Coates, 2010) are examples named as 'grammars'; these centred on the translation of geometrical or spatial modelling to the language paradigm of computing. Here the ambiguity, inherent in encounter with shape and form, needs to be revisited. The component geometric primitives that make up a figure and the way in which they come together matters, not just the way they are perceived. Rules, well understood and applied, could be generative, just as they have been in architecture since the time of the ancient temple builders and beyond. Rules could be applied top-down or bottom-up, exploring emergence. In reality, architectural modelling software is always based on a library of geometrical primitives and operations with clear rule sets that determine and constrain their possible applications and outputs. Syntax is inescapable, not least in the translation of idea from sketch, and natural language to formal language within a computational logic.

Structured software adds levels of syntax that are both language and software specific. Notably, parametric software is generally organised to avoid logical cycles by instituting a clear hierarchical structure. Diagrammatically the graph of dependencies is treelike, directed and acyclic. This dictates clear and early structuring of the relationships between geometrical components. Nodes in the graph can be easily replaced by compatible alternatives; but the logical relationships cannot be reversed, or ambiguous. For instance, if a set of points is defined by being on a set of curves, logically the model must be rebuilt with a new set of points in order to reverse the relationship and create curves that pass through the points. This rigid syntax allows for flexibility in one sense: a large field of possible instances of the geometry complying with the model rules. In another sense, it gives the geometry clear semantic overtones, and to the design as a set of spatial possibilities that are not necessarily inherent in sketching or conceiving of spatial configurations in language. The design workflow is shaped syntactically by the logical structure of the language and software.

In the introduction, I have said that the shift to a modern geometrical paradigm has introduced complexity into the conceptual, operational and constructional spheres of design. It has introduced potential obscurity into the space with certain navigational challenges, to shine light into which, we can look in the same quarter: the history and philosophy of modern mathematics. In the following sections I will explore each of these three spheres in turn through architectural examples.

### 3. The geometrical shift in the three spheres of architecture

**Conceptual sphere: Aesthetic exploration of ideas from mathematics in architecture**

**The modern mathematics of natural systems**

Starting in the conceptual sphere, there has been an appetite for exploring post-seventeenth century ideas from mathematics and physics in architecture in recent decades. This applies to both architectural ideas and expression, and to the act of designing and modelling. Some of these mathematical ideas
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have developed in relation to the analysis of naturally occurring systems and configurations. In interpreting architecture, scholars have long seen the references to natural systems: Laugier’s primitive hut as the origin of the classical orders of architecture (Laugier, 1755); Viollet-le-Duc’s analysis of Gothic structure (Viollet-le-Duc, 1868); Wittkower’s account of Alberti’s abstraction of renaissance astronomical comprehension of the universe to the importance of the circle in architecture (Alberti, 1485; Wittkower, 1952). In the contemporary era it is, amongst other things, the irregularities, the unpredictable detail, the chaotic and the random that have been catching the architect’s attention from the more recent mathematical analyses of phenomena observed in biological and geological systems.

Simulating life

Conway’s game of life, the best-known cellular automaton, is a perfect illustration of an emergent chaotic system. In this case, John Conway honed the rules in order to give a system that would live and propagate in interesting ways over a significant period.

Minifie van Schaik’s Australian Wildlife Health Centre (2006) (Figure 1) uses a cellular automaton to generate masonry set out patterns in the block work. Different outcomes can be accomplished by adjusting the generative rules of the system, altering the colour balance, distribution and relationship to other aspects of the architecture such as wall boundaries and openings. Aesthetically the dynamism of system remains in the built wall, although the living generative system has been frozen at some point in time and masonry (Burry and Burry, 2010).

Notes:

1 Life, or any cellular automaton system, is a deterministic dynamic system with simple rules that can provide abstract models for life, crystal growth, urban growth and many other natural and artificial phenomena. There is no randomness except potentially in the starting state of the system, but the outcomes and progress can nevertheless be impossible to predict except through running the sequence of events in the game. Life for instance is an infinite square grid of cells in which each individual cell can be either alive or dead. The rules are that if the cell has three or more living neighbours it comes to life (is born); if it has two or three living neighbours it stays alive, in any other instance it dies or remains dead.
Mineral aperiodicity

This conceptual experimentation with a mathematical generator is also well illustrated by the unbuilt proposal for Battersea Power Station site by Arup: the Crystal Auditorium (Figure 2a, b). This is another intriguing ‘riff’ on the grid, this time in three dimensions. Combining an idea that the Arup group had explored in earlier projects – Robert Ammann’s aperiodic tiling with a second idea – Danzer’s aperiodic packing developed in response to the discovery of quasicrystals in 1984.

Regarding the Arup proposal for the Crystal Auditorium at Battersea, the design group considered whether the more complex order, exhibited in the quasi crystal could replace the regular orthogonal grid of most building layouts. They had worked with periodic tiling and non-repeating patterns in two dimensions previously and were now interested to extend these ideas into the third. In comparison to the traditional grid with its three sets of intersecting orthogonal grids, this set out grid would have many more planes with less predictable intervals between them. The Golden string was used to create the intervals between the planes: a long and short module in the ratio 1:Phi with each short module flanked by two longs but no predictability about whether two long modules occur together. These ‘Ammann planes’ were then intersected with a set of 15 planes generated as the bisectors of all possible intersections between the planes on the faces of a dodecahedron (12-sided polyhedron). The result was the setting out grid for the building including walls, space-frame for the roof and decorative motifs (Burry and Burry, 2010).

Notes:

Aperiodic tiling patterns are those which cannot be mapped onto themselves by translation. An aperiodic set of tiles allows only aperiodic tilings (as opposed to tile sets that permit both periodic and aperiodic tilings, of which there are many more). In the best known of Robert Amman’s five sets of aperiodic tiles, combining a rhombus and a square tile, the phenomenon known as Ammann Bars can be observed. These are straight parallel lines that run infinitely through the tiling, separated by one of two dimensions in the ratio of 1: Phi (the golden ratio) in a sequence of separations that corresponds to the Golden String. In 1984, Dan Slechtman discovered Quasi crystals with a strange unexplained structure intermediate between crystalline and amorphous. The combination of long-range order and five-fold and icosahedral symmetries observed in the quasi crystals led Ludwig Danzer to arrive at a local mathematical rule base for their structure: tetrahedral polyhedral tiles that are aperiodic with the right face - matching rules. Like the Ammann observation in two-dimension tilings, the Danzer aperiodic packing exhibits parallel planes of infinite extension that run through the packing, conforming to a similar golden string distribution.
Randomness

These two system examples exhibit chaotic and unpredictable patterns – applied in architecture to provide an aesthetic variation operating within a higher-level ordering. Neither of these involves a random component – except, perhaps, for the starting point for the cellular automaton. But the play-off between a degree of randomness on the one hand and order imposed through constraints is applied in architecture, just as it is discovered in space syntactical analysis of settlement pattern. While ‘random’ and ‘design’ may appear at first reading to be near antonyms, randomness can be part of a highly controlled design process. Functions that accept random variables, Voronoi organisation using random points and stochastic approaches to optimisation (finding the fittest solutions for a combination of variables) are familiar examples. Stochastic gives the clue to the relationship – a probabilistic outcome that is not deterministic, that is not precisely predictable, but nevertheless directed or ‘aimed’. Stochastic is derived from a Greek word, which means ‘aim’. A degree of randomness can enrich architecture, just as systems in nature defined as probabilistic lead to diversity within higher-level genetic predictability.

This degree of randomness within a highly constrained geometrical system is something we have introduced into the design of a research project shaped by the interaction of surface and the phenomenon of sound energy. This project

Figure 2b:
Crystal Auditorium, Arup Advanced Geometry Unit (AGU), London 2006.
commenced in response to an intriguing acoustic observation. When the construction of the interior of the Gaudi’s Sagrada Familia Basilica was completed in 2010 and the space cleared of scaffolding and filled with musicians and congregation for the first time, the musicians reported their experience of an extraordinarily diffuse acoustic. Although the vast space has a long reverberation time, the interaction of the reflected sound does appear to result in islands of intensity as in some other monumental stone built interiors. As the interior surfaces are substantially composed of doubly ruled curved hyperbolic surfaces, this led to the speculation that perhaps the hyperboloid of revolution would be a very effective sound scattering surface (consider its shape compared to flat or focusing surfaces, such as spheres, cylinders or cones). Additionally, were this speculation supported (and a passage in Puig I Boada’s, 1976, p.LXXVII) book supports the conjecture that Gaudi was considering the acoustic potential of these surfaces), what would the impact be of altering the shape and orientation parameters of these hyperboloid surfaces if they were arrayed and intersected over a surface in a manner analogous to their application in the geometry of the church? Clearly there would be many factors such as the relationship of the scale of the pattern to the auditory spectrum of sound wavelengths. But it might also be possible to discover some general principles through a combination of digital modelling and simulation, and analogue modelling and testing.

The first experiments were conducted in the Responsive Acoustic Surfaces research cluster at Smartgeometry 2011 in Copenhagen (Burry et al., 2011). One of the experiments overseen by Brady Peters from CITA was a standard controlled test to determine the scattering coefficient over a range of frequencies, for 1:10 scale plaster printed surface models (Figure 3). This testing yielded useful comparative results for different distributions of Z-corp plaster printed discs at 1:10 scale for testing in a diffusion chamber. Comparative results: version built at prototyped at 1:1 scale and Giovanni Betti’s ‘Dolomite’ design.
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Hyperboloids across the surface. The different surfaces tested exhibited very distinct signature profile responses across the frequency range tested. Some general trends conformed to earlier findings such as the relationship between the depth range of the relief in the surface pattern and the frequency range for high scattering values. It was clear from testing the design proposal of cluster participant Giovanni Betti that introducing a degree of variation in the hyperboloid scoops in the surface, a randomness factor, also had the potential to greatly increase the frequency range of effective scattering.

In the Responsive Acoustic Surfaces (RAS) project a full-scale plaster prototype wall was also constructed from hexagonal tiles with molded hyperboloid plaster surfaces (Figure 4). Subsequent research iterations applied and extended these geometrical principles inherited from Gaudi and investigated in RAS to build a more sophisticated and enduring built prototype: the FabPod. This new project moved away from reproducing the surfaces using wet plaster. It explored fabrication and material approaches to reproducing these geometries. This introduced the potential to extend the acoustic ‘tuning’ of the surface to differential material sound absorption and reflection. It also carried with it certain fabrication constraints that I will return to in more detail in the construction section. In particular the aim was to employ sheet materials in the assembly and to achieve a very high fidelity, high quality built prototype. While this would be a prototype in the sense of the first of an anticipated series of experimental spaces to be built, it was expected to have a long and useful life both as a commissioned meeting space and as an acoustic, psycho-acoustic and auditory testing environment.

Figure 4:

Responsive Acoustic Surfaces (RAS).

1:1 scale plaster prototype of identical hexagonal tiles with hyperboloid faces.

(Photo source: Daniel Davis)
It was important for the targeted precision and fidelity in the built artifact that the hyperboloids in the surface should intersect in precisely planar curves. The planes of these curves would define the boundaries between hyperboloid cells. In the general world of intersecting hyperboloids there are few solutions for which this constraint applies (in the general case these fourth order curves are three-dimensional, have points of inflection and would be treacherous to find a good construction solution for). Distributing the hyperboloids on the plane where their intersection curves lie on planes normal to the base plane, or on a sphere with their curves of intersection on radial planes are two special cases (Figures 5 and 6).

Figure 5:
Geometrical solutions for intersecting hyperboloid surfaces with planar intersection curves.
(Source: Daniel Davis and Nick Williams)

Figure 6:
Variations on combining spherical surfaces to create an acoustic enclosure.
(Source: Daniel Davis and Nick Williams)
Constructing hyperboloid surfaces using sheet materials would involve creating molds. The decision was taken to use the same hyperboloid shape surface in all the unique cells of the assembly. Nevertheless there was a desire to introduce a level of variation or randomness into the overall surfaces for two reasons. The first was acoustic: to maximise the frequency spectrum for scattering and to guard against any surprise results brought about by using a regular periodic pattern. The second was aesthetic. The aesthetic agenda was to create a stimulating visual space in which the patterning did not become overbearing through repetition whilst maintaining a lively acoustic for meetings. Acoustically it should be easy to be heard all over the space without raising the voice, with good, even coverage of different pitches in the first reflections and free from the awkward social overtones of an overly quiet ‘dead’ acoustic space (the effect of controlling sound predominantly through absorption).

How could a useful degree of randomness be introduced into the geometrical schema? The answer was that distributing points across a plane would determine the distribution of the hyperboloid cells through a mapping to the hyperboloid centres on the spherical surfaces. This distribution could be regular, irregular, random, or conform to any pattern (Figures 7 and 8).

This is the summary of the geometrically-led ideas in the design of the FabPod schema: 1.) hyperboloids of revolution as sound scattering surfaces, arrayed or distributed across the interior surfaces of a meeting space at a scale appropriate to the scattering of sound in the auditory range and the fabrication constraints; 2.) meeting space form and shape determined through combinations of convex or concave spherical surfaces (an ideal test case as without the hyperbolic treatment these surfaces would have an acute tendency to focus sound and create uneven sound intensity); 3.) variable distribution of similar hyperboloids using a Voronoi pattern, allowing for any pattern including randomisation.

Within the detailed geometrical design of this architecture the genotype/phenotype analogy could be applied at a number of levels once many different versions were generated and tested, for instance schema versus versions of room form, or, room geometry versus hyperboloid distributions.
4. Operational sphere: The challenge of the model space that constructs a problem

The shape of model space

Parametric and programmed computer models for the design of architecture provide a wonderful opportunity for rapid versioning and feedback, whether in very early design for the development of ideas in response to a design brief or throughout the refinement and better acquaintance with the facets of a particular design direction. Conversely moving beyond three dimensions, the model space also moves beyond what is easily visualised or conceptualised. This is a direct implication of the transition to the analytic geometry of Descartes and Fermat – the adoption of modern geometric thinking. The system cannot be represented in three dimensions – it has too many degrees of freedom. The model has a shape of its own. The singularities and bifurcations in the shape of this space to which Deleuze (1993), Cache (1995), and Migayrou (2006) have alluded are not metaphorical, they are palpable, yet visualisable only through the visualisation of change. This is not new space – we can find similar spaces in the models of biological and embryological systems and processes. We see their mathematical treatment in the writing of Thom (1975). They are newer to design. In

Figure 8:
The built FabPod wall showing the distribution of the hyperboloids, material treatments and the cell reflection rule across the junctions between spherical surfaces.
science their complexity is constrained by that which can usefully be computed for analysis. In design they may be purely synthetic and may grow to any order of complexity (or complication) until constrained by their own brittleness to serve no more. While each model may represent many design solutions, it is nevertheless a highly defined and constrained design domain. It may be one particular path amongst many approaches with limited opportunities to backtrack or move laterally within the same model.

The engagement with this space is not only synthetic for the designer constructing it but also interrogative. This space can be altered and refined steadily but only in response to understanding its possibilities, limits and extents. How do you interrogate such a space if it cannot yield to visual inspection or visualisation, and you can only view isolated instances in a field of possibilities? In general, architects still work through the selective targeting of isolated instances and in reaction to viewing and responding to create the next instance or version or alter the model. Why assume that visualisation is a significant way to apprehend space? Geir Kaufmann (1980, p.115), examining the imagist position with regard to thought in a time of dominance of information processing and linguistics in psychology models of thought concludes: ‘Visual imagery is particularly suited for the execution of transformational activity needed in tasks with a high degree of novelty.’ Transformational activity with a high degree of novelty sounds a lot like design.

**Palpation**

In computer science parlance, the relationship between design intentions and those intentions expressed as a map of geometrical relations in a parametric design model schema is non-obvious. Similarly, the relationship between the design schema as a diagram or map of the model and its relationships and parameters and the actual ranges of the parameter values is also non-obvious. In other words these things are not easily seen, they are not open to view; not plain, manifest, clear, palpable and certainly not unmistakable. In addressing the question of ‘designerly ways of knowing’ more complex digital geometrical model spaces, leaving aside, for the moment, the question of whether and how we can see or visualise the ‘shape’ and extents of the geometrically constructed model space, let us turn to whether and how we can palpate that which is not palpable (Burry, 2010).

One way is to explore it by steps until encountering the boundaries. Test it through incremental change until it fails. Where it is driven by quantifiable outputs in relation to inputs (best natural energy use for greatest glass area), it exploits the indifference of the computer itself to the tedium of searching and comparison in order to find better and optimal solutions. This provides some, albeit partial, empirical knowledge of the complex boundaries, navigable expanses and holes in the space of the model that feed back into its intuitive spatial exploration and manipulation (ibid).

**Model space syntax**

In reality the space syntax, the high level symbolic organisational diagram of the logic of the geometrical relationships in the model often plays a much more significant role in apprehending its nature and operating it than any embodied experience of the model space that is being/has been created. The relationship between the diagram and the space is one of cause and effect.

Similarly, computational models can be highly automated in their generation, self-populating through relatively economical coded commands including conditionals and stopping plans. But such models only permit human intervention at a very high level (adjusting the program or script and rerunning). What of the architectural model that has a well defined generative geometric
pattern or hierarchy of more hybridised geometrical relationships but must permit the intervention of the human hand on an ad hoc basis, the occasional breaking of rules?

In modelling for the realisation of the Sagrada Família Basilica we have encountered both paradigms. Notably the bone-like colonnade on the Passion façade exhibits a clear geometrical schema and relations between the highly differentiated component parts (the columns themselves, the naturalistic ramping landscape in which they sit, the hexagonal prism frieze above and the stepping cornice above). They conform to parabolic setting out in plan and growth sequencing in elevation. But over many iterations and refinements of the design solution, only by integrating certain local hand wrought changes and irregularities in the model, could a satisfying solution retaining the animation and power of the Gaudí drawing guiding the endeavour be accomplished (Figure 9).

At the other end of the spectrum there are architectural models that are literally ‘constructed problems’ in analytic geometry. Chris William’s 2001 solution for the Foster + Partners dome design for the British Museum is an example (Williams, 2001; Burry and Burry, 2010) (Figure 10). The geometry of the irregular site and supports for the dome, the structural support constraints and the relaxation of the nodes for the triangular panels over the surface are all encapsulated in the mathematical description of the surface. One of the 11 un-built Australian works exhibited in Australia’s virtual pavilion at the 2014 Venice Biennale, the Hybrid Cathedral by Tessellate, pursues this same notational economy through editing the algebraic description of a complex undulating surface that becomes the structure, separating the monumental interior from the housing and commercial spaces created on the exterior in the surface folds (Figure 10). These projects are examples of operational modelling using high-level mathematical descriptions.

Returning to the FabPod project, this was a commissioned acoustic meeting space within a large open office area. The aim was to design a partial enclosure (open at the entry and to the ceiling) that would comfortably accommodate eight people around a table, reduce sound transmission as closely as possible to the values ascribed in the Standards for a fully enclosed prefabricated meeting room, and create a well tuned internal acoustic and hushed immediate external acoustic.

Figure 9:
Passion façade of the Sagrada Família Basilica. Hybrid geometries following a hierarchy of rules and incorporating haptic craft in the digital modelling.
As part of the design research, multiple authors were involved in the creation of the digital modelling workflow. They should be able to work in different environments suited to different tasks and stages, with appropriate protocols for exchanging information and data at the interfaces between individuals and software (Williams et al., 2013) (Figures 12 and 13). This system went beyond an architectural model.
Figure 12:
Computational of the FabPod.
(Source: Daniel Davis)

Figure 13:
Acoustic simulations of the FabPod in the space.
(Source: Brady Peters)
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– it was essentially a framework, clearly mapped from top down, with the component parts. Through this design for seamless integration of the component contributions, the issues with the design for the detailed fabrication of individual cells could be dealt with separately from the population of spherical surfaces by hyperboloids (a computationally heavy Boolean operation).

As a space syntactical topological graph, this project includes parallel development, undirected links and cycles, with feedback from any part of the system able to influence the design of any other part until the first physical component is cut. I believe that this can legitimately be viewed as an approach both to architectural modelling and to navigating the model. It deals with the issue of sole author ‘lost in design space’, the opacity of the model to all outside the modelling process (Burry and Burry, 2008) and breaks down the project complexity, reducing organisational and navigational risks but without relinquishing the design flexibility and design navigability throughout the process.

5. Constructional sphere: The challenge and opportunities of constructing the less geometrically constrained
The introduction of randomness, or any degree of continuous variation or unpredictability in architecture generally implies greater complexity in the steps of translation to physical artifact. Or perhaps rather than ‘physical artifact’, we should consider this the ‘tectonics of spatial boundaries’ to relate it more closely to space syntactical thinking. It is at this stage of translation that digital computation is now proffering some of its greatest gifts to architecture. High level geometrical system rules can be mediated to instruct the operation of a computer numeric controlled machine that may attend to information as basic as 3D coordinates for the deposition of a unit of plaster or ABS plastic.

For the FabPod project taken as a case study in the previous sections, the compound angles in which the framing components for each cell met ensured that two rotational angles for the cutting tool head also needed to be added, and a more sophisticated 5-axis router deployed. Nevertheless, in principle the range of CNC technologies that can accept information more or less directly from the digital model (with grateful acknowledgement of those who work to integrate the middleware to make the translation to machine code from the design environment more streamlined) means that, depending on scale and materiality, shape need not be the cost and complexity barrier it once was. Moreover, returning to the phenotype paradigm, the opportunity to generate multiple offspring or vari-
ants from one schematic model, not just as a digital outcome but as an affordable constructed outcome is ever more accessible. FabPod belongs to a genre of prototypes, many developed in academic research environments, that demonstrate that there is scope to automate the fabrication of phenotypically unique but familial components. The current labour intensive bottleneck remains the assembly of components into subassemblies and construction. But this is not inevitable – the widespread uptake of robotics in architectural construction offers the opportunity to program variation into the assembly line also.

In the FabPod project, the generic construction problems of over-acute angles, under length cell edges, boundary conditions, and node matching across surface boundaries were all tackled within the environment for the creation of the base geometry. Downstream, this schematic model with all the parent geometry defined, created and programmed in Grasshopper for Rhino, was passed along with the model of the generic cell construction geometry from another source, to team members expert in CATIA to automate the digital build of the actual cells, adding all the component geometry (timber-based framing elements with 5-axis angle cuts etc.). This was the environment for clash detection and subsequent unrolling of the components to provide the cutting layouts and tool paths for the fabrication of the components. In part, the versatility of the Agile approach was attributable to the overlaps in the system phases and processes. Here physical construction began in earnest in a fully virtual environment, prototyping the every detail of the physical assembly with material components and tolerances.

Even throughout construction, there was a great deal of further experimentation, just-in-time decision making, prototyping and feedback regarding the detailed material systems, fabrication techniques, sequencing and construction. But there was a clear boundary between full design geometry flexibility and construction detail flexibility. From the first physical cut onwards, there could be no change to the geometry, only the material means and techniques of realising it.

6. Discussion and conclusion

The common use of the term ‘construct’ for almost any abstract structure, such as, concepts, [architectural] theories, systems, worlds and ‘the world’, is an index of Kant’s philosophical triumph. Significantly, it is ‘the outcome of a signal alteration in the way mathematics itself is practiced and understood in the early modern, pre-Kantian period’ (Lachterman, p.viii, 5). In architecture, ‘construct’ has also taken on a new, more abstract meaning. Previously models were constructed to represent the designed work or all that was so far known about it – the current state of the design. Now model construction is the construction of the design as a process or system that nevertheless requires sampling to extract instances from the world of abstract logics and syntax, back into the classical geometric paradigm. Only here are they accessible to the ‘pure intuition’ that Kant gives us as the basis of our apprehension of space. However, Kant makes a definitive division between synthetic knowledge and analytic knowledge. The knowledge of space is a priori and synthetic. Analytic knowledge is derived from logical deduction. Therefore a question mark seems to exist over spatial concepts in mathematics that are derived from logic and are outside an immediate intuitive apprehension of space and geometry. For Kant, geometry, like properties of space it defines, is ‘pure a priori intuition’.

This brings us back to Hillier and Hanson’s ‘socially constructed’ space, its conceptually constructive nature owing a debt to Kant. To what extent is it spatially, or essentially synthetically, derived, or to what extent analytically derived, or the product of pure logic? It could be argued that we can use modern mathematics to dissemble the spatial into
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pure logic, a state in which it becomes subject to, rather than generative of, its syntax. Part of this magic, is that it is possible to visually or cognitively reconstitute logically derived structures in ways accessible to spatial apprehension. This deft translational possibility between the two worlds of modern and classical geometry seems to be at the heart of the relationship between design and computation.

This paper has argued that in architecture there has been a more recent shift from a classical to a modern approach to mathematics, geometry and space. This is reflected in much more than the immediate preoccupation with the aesthetics of ideas appropriated from mathematics and the sciences. It is the shift from constructing geometry to constructing the problem. This approach submerges designers in more complex system spaces – real spaces that cannot be immediately imagined or envisioned by recourse to three-dimensional geometry but require other approaches to navigation. This paper considers these navigational challenges in the conceptual, operational and constructional spheres in architecture, reflecting on the links to space syntax.

The examples and case study have revealed a number of generic approaches to the navigation of design- or model- spaces at different levels of abstraction. At the extremes these might be characterised as palpation and open networks over a set of overlapping modules. The first of these is an exploration from within: exploiting synthetic intuition of the spatial. The second is a mapping from without: exploiting the analytic organisational opportunities of logic.

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